Technical white paper: TE326

Interferometric angle measurement and the hardware options available from Renishaw

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Introduction

This paper describes the operational principles behind Renishaw's interferometric angular optics and how they can be used to measure pitch or yaw errors in a linear axis, the angular accuracy of a rotary axis, or surface flatness. It examines the various error sources that can affect the accuracy of measurement and the various angular optics hardware options available from Renishaw and how they have been optimised to address these error sources.

Angular optics - operational principles

Figure 1 shows a typical setup for measuring the yaw error in the linear motion of the X axis of a moving table machine. The three key components are:

- The laser (shown mounted on a tripod)
- The angular interferometer (shown attached to the spindle)
- The angular reflector (shown mounted to the moving table)

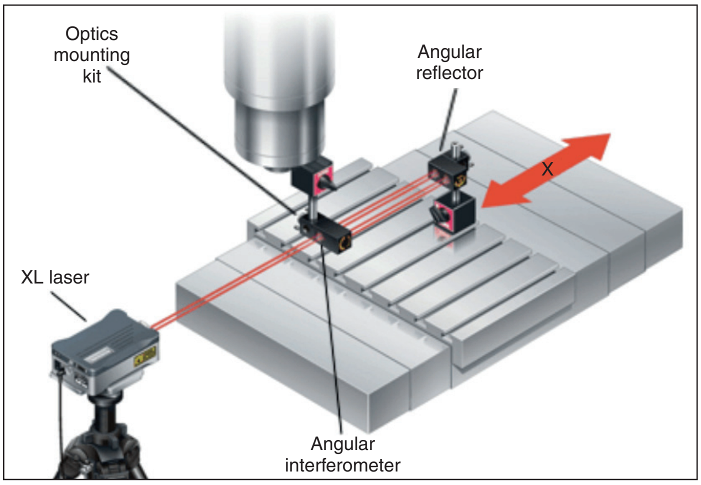


Figure 1

As the table moves in the direction of the arrow (X axis), the laser and optics will measure any yaw (twisting) error in the table's motion. Alternatively, if both optics are rotated 90° about the beam axis, then the same optics may be used to measure any pitch (tipping) error in the table's motion. Figure 2 shows a close-up view of the optics in this orientation.

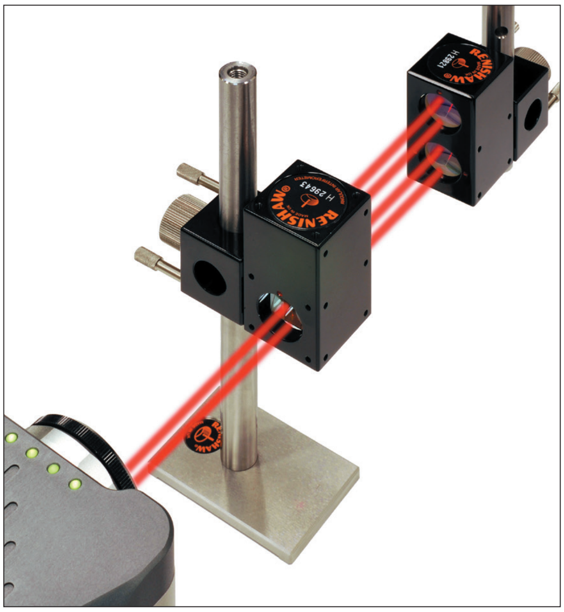


Figure 2

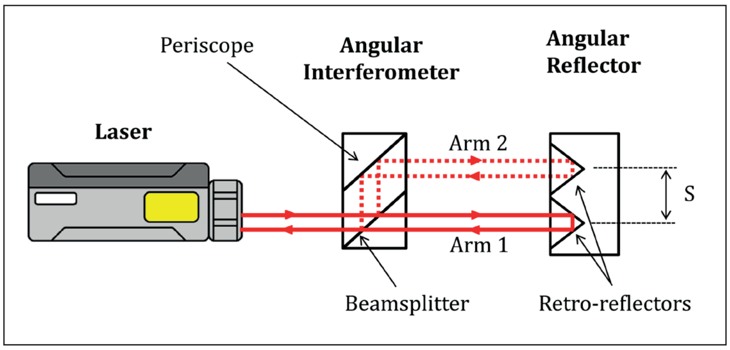


Figure 3

In order to understand how the angular interferometer works, it is necessary to examine the beam paths within the optics, as shown in Figure 3. In this section it is assumed that the optics are perfectly aligned. The angular interferometer contains a combined beam-splitter/periscope optic and the angular reflector contains two retro-reflectors with a centre to centre spacing of S. When the output beam from the laser reaches the angular interferometer it is split into two separate beams by the internal beam-splitter. One beam (solid red) continues straight on to the lower retro-reflector and forms “arm 1” of the interferometer. The other beam (dotted red) is reflected upwards by the beam-splitter and then reflected by the periscope mirror out to the upper retro-reflector to form “arm 2” of the interferometer. Both beams are then reflected by the retro-reflectors back the way they came. When they reach the angular interferometer they are recombined before travelling back to the laser head detection unit where they interfere to produce a measurement signal.

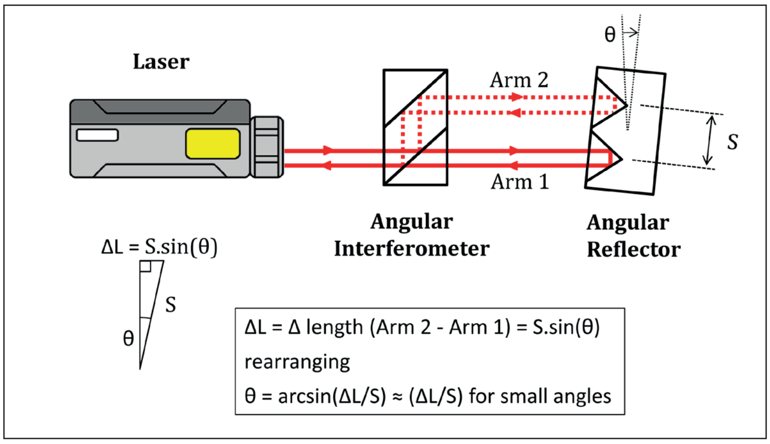


Figure 4

The laser system measures changes in angle by detecting the relative changes between the optical path lengths in the two “arms” of the interferometer (ΔL). Consider what happens if the angular reflector is pitched (tipped) away from perfect alignment by an angle θ, as shown in Figure 4. As the optic tips, the beam in Arm 1 will get shorter by ½.S.sin(θ) as it travels to the lower retro-reflector. At the same time the beam in Arm 2 will get longer by ½.S.sin(θ). Thus making a relative change in the outward path lengths, between Arms 1 and 2 of the interferometer, of S.sin(θ). Note that because the beams travel back and forth between the angular interferometer and reflector, the total path length change is doubled to 2.S.sin(θ). This change in path lengths is detected by an interference fringe counter/interpolator inside the laser’s detector unit and converted into a linear distance ΔL by multiplying by the laser wavelength/2.

$$\Delta L = \text{Fringe count} \times \text{laser wavelength} / 2$$

In angular mode the laser system software converts the relative change in path lengths, ΔL, into an angle by calculating arcsin($\Delta L/S$), where S is the separation between the retro-reflectors.

$$\theta = \arcsin(\Delta L/S)$$

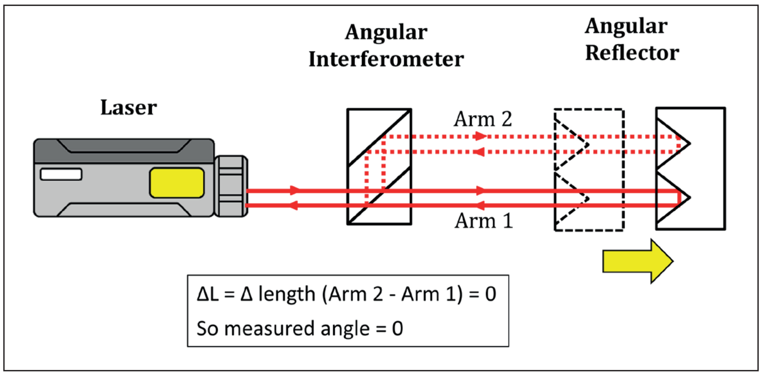


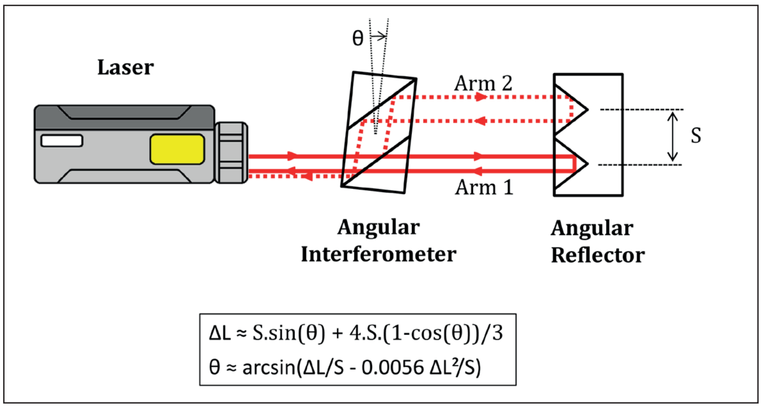
Figure 5

Now consider what happens if the angular reflector is moved along a linear axis, without being pitched (tipped), as indicated by the yellow arrow in Figure 5. As the reflector moves, the beams in Arms 1 and 2 of the interferometer will both get longer at the same rate. Therefore the relative change in path lengths between Arms 1 and 2 of the interferometer will be zero, and there will be no change in the displayed angle. The same is true if the angular reflector moves in a direction perpendicular to the beams by a small amount (i.e. not enough to misalign the laser beam and cause a loss of signal). In this case both beams are affected equally so there is no change in the displayed angle.

It can also be shown that the angular interferometer is largely insensitive to small changes in the alignment of the laser relative to optics. The angular measurement depends almost entirely on changes to the relative angle between interferometer and reflector optics.

The sensitivity of an angular interferometer to change in angle between interferometer and reflector, whilst being insensitive to linear translation and the precise alignment of the laser, makes it extremely useful for calibrating the accuracy of rotary axes, and for detecting pitch and yaw errors in the motions of linear axes.

The analysis so far has concentrated on the case where the reflector is the moving optic. However, there are occasionally times when it is easier to arrange the optics so that the interferometer is the moving optic. Consider what happens when the interferometer (instead of the reflector) is pitched by an angle θ, as shown in Figure 6.



$$\Delta L \approx S.\sin(\theta) + 4.S.(1-\cos(\theta))/3$$
$$\theta \approx \arcsin(\Delta L/S - 0.0056\,\Delta L^2/S)$$

Figure 6

The relative change in the path lengths of Arms 1 and 2 as they pass through the air between the interferometer and reflector is the same as before but with the opposite sign (i.e. -2.S.sin(θ)) since when the interferometer is tipped Arm 1 now gets longer and Arm 2 gets shorter. However, the overall change in path length of Arm 2 is now more complex than before because the path taken by this beam within the glass periscope has also changed. This causes a small increase in the path length in Arm 2 of approximately 2.S.n(1-cos(θ/n)) where n is the refractive index of the glass. This gives an overall total relative path length change between arms 1 and 2 as they travel back and forth of approximately -2.S.sin(θ) + 2.S.n(1-cos(θ/n)). Because this additional change in path length is not accounted for in the laser system software, a small measurement error is introduced. The magnitude of this error is shown by the table in Figure 7.

The table shows that, when using the laser to measure pitch or yaw errors of less than ±0.1° in a linear axis, the additional measurement error introduced by using the angular interferometer (rather than the angular reflector) as the moving optic is insignificant. However, when measuring larger angles (e.g. when calibrating a rotary axis) the angular reflector must be the moving optic unless additional measures are be taken to mathematically correct the results. It should also be noted that the angular measurement range is significantly reduced if the interferometer is the moving optic because the return beam from Arm 2 becomes misaligned (see Figure 6) more quickly as the angle is increased.

Measurement errors introduced by using moving interferometer	
Angle measured (degrees)	**Measurement Error (%)**
0	0.00%
0.01	0.006%
0.1	0.06%
1	0.6%
2	1.2%
5	3%
10	No signal

Figure 7

Angular optics - typical applications

Angular optics can be used for a variety of metrology applications, for example;

- Measurement of pitch or yaw errors in linear axes of motion, as illustrated by Figure 8 which shows the measurement of a pitch error in the X axis of a moving table machine. The pitching of the table as it moves (shown dotted) has been grossly exaggerated for clarity.
- Calibration of rotary axes in combination with a counter-rotating high accuracy angular reference such as XR20-W. For example Figure 9 shows the calibration of a rotary table using an angular interferometer in combination with the XR20-W angular reference. As the rotary table is rotated to each target position, the highly accurate XR20-W angular reference is counter-rotated to maintain alignment of the angular reflector to the interferometer, allowing the laser to measure the rotational errors in the rotary table relative to the accurate reference. This application is covered in more detail in technical white paper TE327 *'Interferometric calibration of rotary axes'*.
- Measurement of guide-way straightness or table flatness by taking a sequence of angular measurements using a small sliding base with supporting feet of known separation, together with angle to straightness or flatness measurement software. Figure 10 shows an angular interferometer being used in combination with the flatness measurement kit to assess the flatness of a granite table. A series of angular readings are taken along eight measurement lines, using a straight-edge as a guide, and then combined by the software to produce a flatness plot similar to that shown.

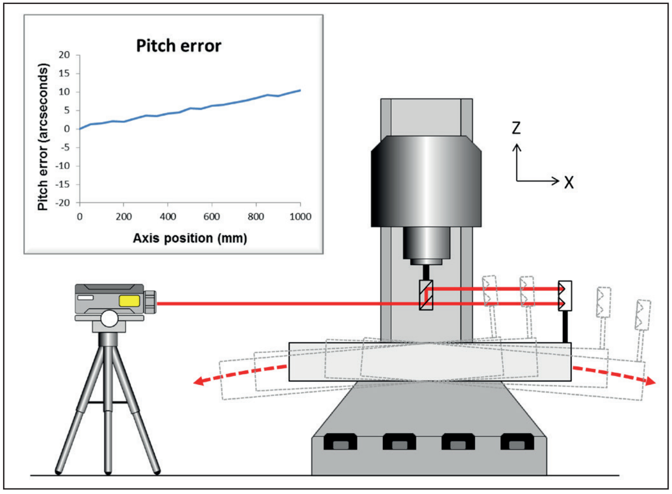


Figure 8

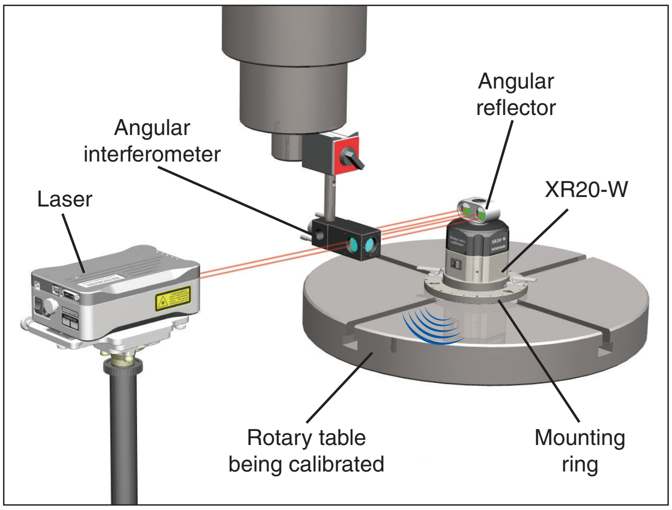
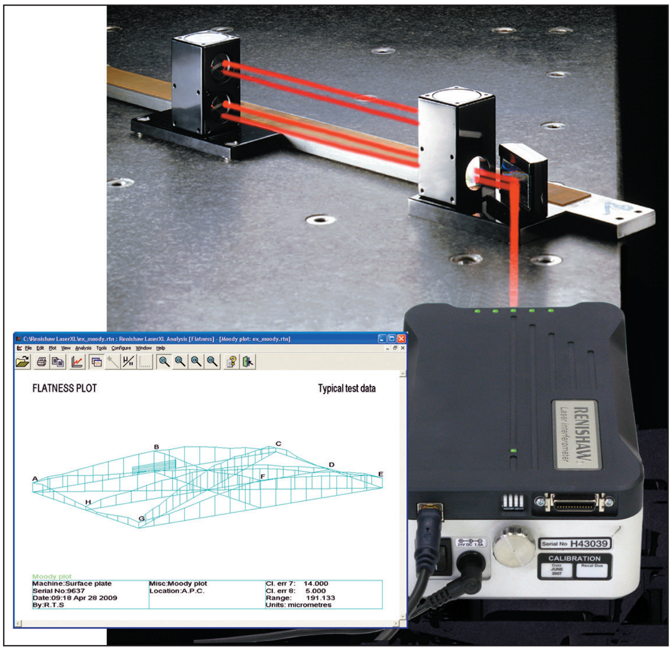


Figure 9



Machine:Surface plate
Serial No:9637
Date:09:18 Apr 28 2009
By:R.T.S

Misc:Moody plot
Location:A.P.C.

Cl. err 7: 14.000
Cl. err 8: 5.000
Range: 191.133
Units: micrometres

Figure 10

Limitations on accuracy - the sources of measurement error

This section describes the main error sources that affect the accuracy of interferometric angle measurements. It also indicates steps that can be taken to reduce these errors. This analysis concentrates on the case where the reflector is the moving optic.

Incorrect retro-reflector spacing

The equations used by the angular software to convert the relative path length difference, ΔL, between arms 1 & 2 of the interferometer, rely on an accurate value for S, the centre to centre separation between the two retro-reflectors in the angular reflector. For standard Renishaw angular optics the separation between the retro-reflectors is 30 mm. A value for S of 30 mm is therefore predefined within Renishaw's angular software and this value is used, by default, to convert ΔL into the measured angle. The accuracy of the value of S is critical as it has a direct impact on the accuracy of the angular measurement, θ. For example, if there is a 1% difference in the actual separation between the retroreflectors and the value of S used by the software, the error of measurement will be almost exactly 1% of the angle being measured. There are three possible sources of error here.

- Manufacturing tolerances on dimension S
- Thermal expansion/contraction of the angular reflector
- Incorrect configuration of "Angular factor"

These error sources are considered in turn.

Manufacturing tolerance. Considering Renishaw's standard ±0.2% accuracy angular optics, manufacturing tolerances can introduce a variation in the centre to centre spacing between the retroreflectors of up to 30±0.06 mm. In percentage terms this variation is (±0.06/30) x 100 = 0.2%. All standard reflectors are tested to ensure they will meet the ±0.2% accuracy specification over the operating temperature range (0-40 °C).

Alternatively, Renishaw's angular measurement software allows the user to enter an "angular factor" (K), in the configuration menu. By default K=1.000. However, if the value of K is set to something other than 1.000, then the software applies a slightly modified equation when calculating the measured angle of either, $\theta = \arcsin(\Delta L/(K.S)$ if arcsin correction is enabled, or $\theta_{radians} = \Delta L/(K.S)$ if Arcsin correction is disabled. This allows the system to apply a correction if the exact separation of the retro-reflectors is known. If, for example the retro-reflector spacing was 30.3 mm instead of the nominal 30 mm, then K could be set to 1.01 to correct for this.

Renishaw can supply calibrated angular optics with a certificate stating the angular factor that should be entered when using them, thereby allowing angular measurement accuracies approaching 0.02% (200ppm) to be achieved, depending on the environment.

Note: When calibrating rotary axes using XR20-W, Renishaw's Rotary axis calibration software includes an automated angular optics calibration procedure which automatically identifies any discrepancy in the actual separation between the retro-reflectors, and then adjusts the value of the angular factor, K, accordingly, to eliminate the error.

Thermal expansion or contraction of angular reflector. The spacing between the retro-reflectors is determined by the metal housing they are mounted in. If the temperature rises, the housing will expand, and the retro-reflector spacing will increase, and vice-versa. Fortunately, this effect is relatively small. For Renishaw's standard Angular optics, the housing is made from Aluminium, (which has a linear coefficient of thermal expansion of 23ppm/°C), and the retro-reflector spacing is nominally 30 mm at 20°C. Therefore, the retro-reflector spacing will increase by about 0.7µm for every 1°C increase in temperature above 20°C. This in turn will cause an error in the measured angles of +23ppm (0.0023%) for every °C rise above 20°C.

Suppose angular measurements are taken at an ambient temperature of 30°C. The measurement error introduced by expansion of the reflector housing will be +0.023% of the angle being measured. This error can be ignored when measuring pitch and yaw errors in linear axes. (The pitch and yaw errors in linear axes are typically <100 arcsec, making the additional error in the measurement due to thermal expansion at 30 °C less than 0.023 arcsec, i.e. negligible).

However, when measuring large angles with calibrated optics, this error source becomes significant. Fortunately, when calibrating rotary axes using XR20-W, Renishaw's Rotary axis calibration software automatically takes any expansion or contraction of the reflector housing into account. This is because the automated angular optics calibration procedure will identify the discrepancy in the actual separation between the retro-reflectors at the current temperature, and then adjusts the value of the angular factor, K, accordingly. So, providing the temperature of the reflector remains stable within ±1°C during the subsequent calibration of the rotary axis, any remaining error due to reflector housing expansion will be insignificant.

If large angles are being measured with calibrated angular optics, without XR20-W, then the angular factor supplied with the optics may be modified according to the temperature and expansion coefficient of the reflector housing material using the following equation.

$$K_T = K_{20}(1 + \alpha\ (T\text{-}20))$$

Where:-

K_{20} = angular factor supplied with the calibrated optics (calibration temperature = 20°C)
α = linear coefficient of expansion of reflector housing
T = current reflector housing temperature in °C
K_T = K value that applies at current temperature and must be entered in the software

For example, suppose a set of standard Renishaw aluminium housed angular optics are supplied with a calibration certificate showing K_{20} = 1.050000, but are going to be used at 25°C to calibrate a rotary axis without using XR20-W. The value for K_T that must be entered as an "angular factor" in the software is $1.05000(1 + 23 \times 10^{-6}.(25\text{-}20)) = 1.050121$ (rounded to 6 decimal places).

An alternative approach to reducing this error when measuring large angles without XR20-W is to use optics with housings made with material of a lower expansion coefficient. Renishaw can supply retro-reflector optics mounted in Stainless Steel (α = 10ppm/°C) or Invar (α = 1.2ppm/°C).

Incorrect angular factor setting. It's important the user configures the angular factor (K) correctly. If the factor is entered incorrectly, the measurement error as a percentage of the angular reading is given by Error (%) = $100 \times (K_{incorrect} - K_{correct})/K_{correct}$.

Arcsine correction

As described previously, the correct equation for converting the change in relative path lengths, between arms 1 and 2 of the interferometer, into an angular readout, is $\theta = \arcsin(\Delta L/S)$, where S is the separation between the retro-reflectors and ΔL is the change in relative path length.

However, early laser interferometer systems utilised the simpler radian approximation equation, $\theta_{radians} = \Delta L/S$, because it was easier to implement in the hardware available at the time, and

Measurement error if Arcsine correction is not enabled			
Angle being measured (degrees)	**Measurement error**		
	(degrees)	**(arcseconds)**	**(as %)**
0	0.0000	0.000	0.000%
0.1	0.0000	0.000	0.000%
0.2	0.0000	-0.001	0.000%
0.5	0.0000	-0.023	-0.001%
1	-0.0001	-0.183	-0.005%
2	-0.0004	-1.46	-0.020%
5	-0.006	-22.8	-0.127%
10	-0.051	-182.5	-0.507%

Figure 11

had a negligible effect on accuracy when measuring small angles. The table in Figure 11 shows the measurement error introduced by using the radian approximation. Even at an angle of 5° the error is only ~0.1%, which is half the ±0.2% accuracy specification of Renishaw's standard angular optics.

Renishaw's angular measurement software allows the user to select whether Arcsin correction is applied or not via a configuration menu. If "arcsin correction" is disabled the angular software uses the radian approximation equation, thereby allowing direct comparison with readings taken with older laser systems. If more accurate measurements are required "arcsin correction" must be enabled using the configuration menu in the software.

Misaligned reflector at datum (i.e. not perpendicular to laser beams)

The equation for converting the change in relative path lengths, between arms 1 and 2 of the interferometer into an angular readout, of $\theta = \arcsin(\Delta L/S)$, assumes that the apexes of the retro-reflectors within the reflector lie on a line that is perpendicular to the laser beams when the laser system was datumed, so that S and ΔL lie on two sides of a right-angled triangle. However, if the reflector is not perpendicular at datum, this is no longer true, the triangle is no longer right-angled and this will introduce a small measurement error when ΔL is converted to an angle using either the arcsin or radian approximation equations.

Suppose the laser system is datumed with the reflector misaligned by a small angle θ_0 as shown (grossly exaggerated) in Figure 12. The system is then used to measure an angle of θ. Basic trigonometry shows that under these conditions the correct relationship between the change in the relative path lengths of Arms 1 and 2 (ΔL) and θ, is now $\theta = \arcsin(\mathrm{Sin}(\theta_0) + \Delta L/S) - \theta_0$. However the angular software uses the equation $\theta = \arcsin(\Delta L/S)$, thus introducing a small measurement error.

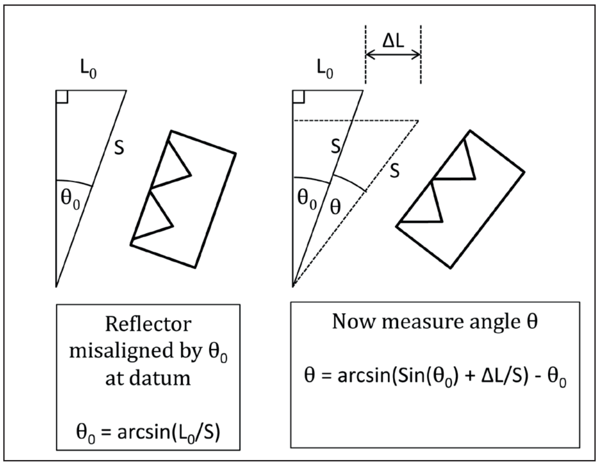


Figure 12

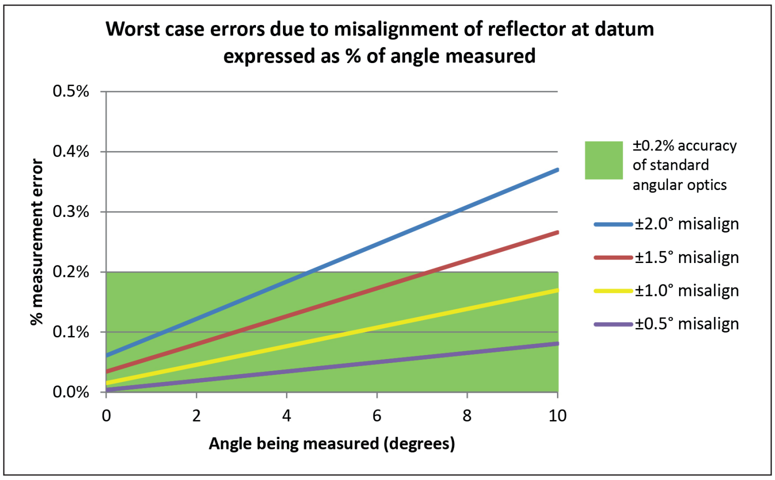


Figure 13

Figure 13 shows how the measurement error (expressed as a percentage of the angle being measured) varies with reflector misalignments and the angle being measured, with arcsine correction enabled.

If measuring small angles (e.g. pitch and yaw errors in a linear axis) with standard ±0.2% accuracy angular optics, the additional error due to misalignment of the reflector at datum starts to become noticeable (>±0.05%) when the reflector is misaligned by more than ±2.0°. Alignment within this level is usually possible to achieve with the naked eye.

If measuring larger angles, or working to higher accuracy levels with ±0.02% calibrated optics, it is recommended to align the reflector using a mirror or slip gauge held against the front of the angular reflector housing, whilst the reflector is adjusted so that the laser beam is reflected back into the laser's output port.

Note: When calibrating rotary axes using XR20-W, Renishaw's Rotary axis calibration software includes an automated angular optics calibration procedure which automatically identifies any misalignment of the reflector and then mathematically corrects all subsequent readings to eliminate the error.

It should be noted that similar misalignments of the angular interferometer do not cause the same type of error. However, the interferometer should be aligned to be perpendicular to the laser beam within ±2° to maximise interference signal strength and purity and avoid sub-divisional errors (see later section). Again a slip block or mirror can be helpful here.

Different depth retro-reflectors

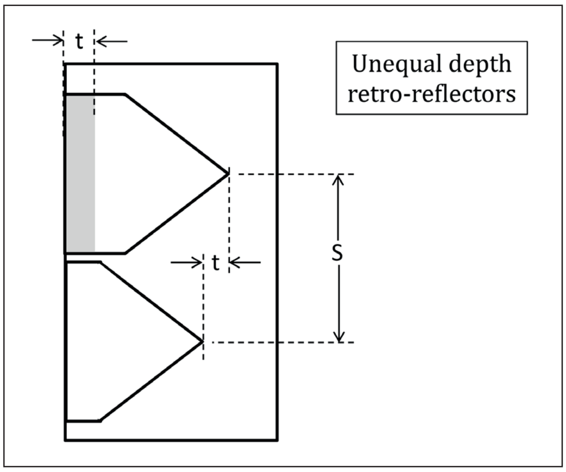


Figure 14

A similar type of error occurs if the retro-reflector depths are not equal, due to manufacturing tolerance variations. Figure 14 shows an angular reflector containing two retro-reflectors with a depth difference of t between them. In this case, even if the front of the angular reflector housing is set perpendicular to the laser beams when the laser system is datumed, the line between the apexes of the retro-reflectors will not be perpendicular. This misalignment introduces a similar form of error to that caused by datuming the laser system with the reflector housing misaligned by a small angle (as described in the previous section). Analysis shows that, with a mismatch in retro-reflector depths of t, the relationship between the change in the relative path lengths of Arms 1 and 2 (ΔL) and θ, is now almost exactly $\theta = \arcsin(\mathrm{Sin}(\theta_0) + \Delta L/S') - \theta_0$ where $\theta_0 = \arctan(t/(S.n))$, $S' = S/\cos(\theta_0)$, and n is the refractive index of the retro-reflector glass ≈ 1.5. However the angular software uses the equation $\theta = \arcsin(\Delta L/S)$ to calculate the displayed angle, thus introducing a small measurement error.

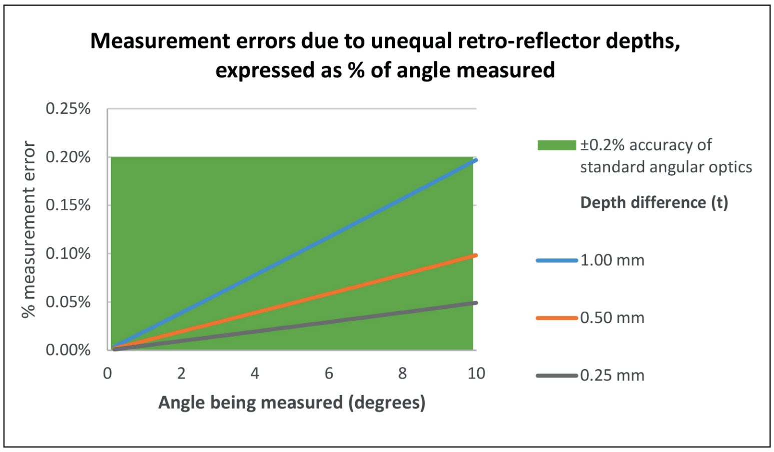


Figure 15

Figure 15 shows how this measurement error (expressed as a percentage of the angle being measured) varies with the difference in retro-reflector depths and the angle being measured.

The manufacturing tolerance of Renishaw's retro-reflector optics ensures that, even when measuring angles up to ±10°, the error due to unequal retro-reflector depths is kept within ±0.2%, the accuracy specification of standard angular optics. If measuring small angles, (for example pitch and yaw errors in a linear axis), the error is insignificant.

However, when using higher accuracy calibrated angular optics (±0.02%) or, if calibrating larger angles to higher accuracy levels, then the errors introduced by unequal retro-reflector depths can become important. Usually such measurements are made in combination with Renishaw's XR20-W and Rotary axis Calibration Software. This system includes an automated angular optics calibration procedure which automatically identifies any misalignment of the angular reflector and variation in retro-reflector separation and then mathematically corrects all subsequent readings to eliminate the error.

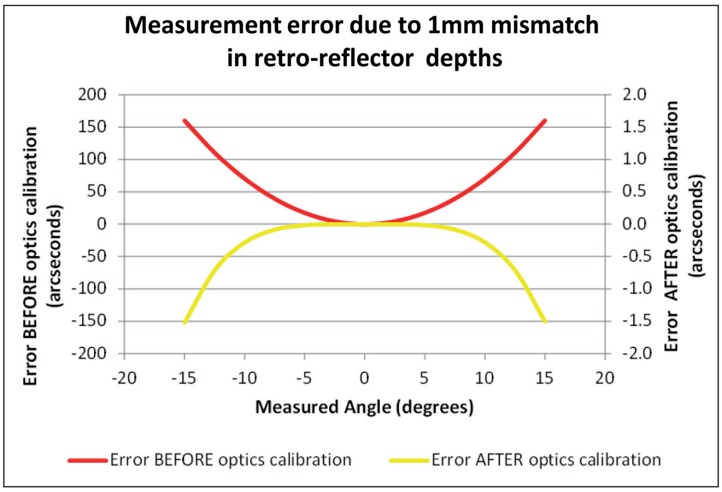


Figure 16

Although the form of error caused by unequal retro-reflector depths is not absolutely identical to that caused by a misaligned reflector, it is sufficiently similar that the same calibration procedure also largely eliminates this error over the calibrated range of ±5°, as illustrated by Figure 16. Note that the measurement error before calibration is plotted in red against the left hand axis, and the error after calibration is plotted in yellow against the right hand axis. The optics calibration routine has reduced the error by a factor of over 100 over a ±15° range, and by even more over the central ±5° range where calibration was performed.

Non-parallel beams from angular interferometer

If the beam-splitting interface and the periscope mirror in the angular interferometer are not perfectly parallel, then the beams emerging from the periscope, (and hence the beams in Arms 1 and 2 of the interferometer) won't be parallel. This is shown (grossly exaggerated) in Figure 17, where the beam in Arm 2 is shown misaligned to the beam in Arm 1. Renishaw's interferometer optic is manufactured to very high tolerances, and the beams emerging from the interferometer are guaranteed* to be parallel to within ±15 seconds of arc.

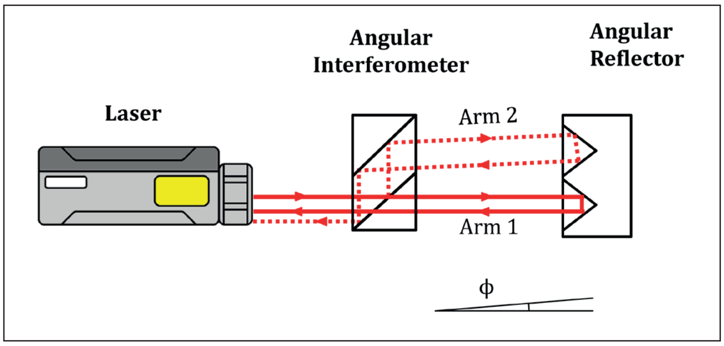


Figure 17

**The tight parallelism tolerance is an essential requirement which ensures the optics can be used to measure the pitch or yaw errors of linear axes up to 15m long without loss of signal strength due to excessive misalignment of the returned beams.*

Nevertheless, even with beams that are parallel to within 15 arcsecs, there is a small effect on measurement accuracy.

Firstly, consider the effect when using the optics to measure large angles, such as when calibrating a rotary axis. Detailed optical modelling of the system shows that if Arm 1 is perfectly aligned, and Arm 2 is misaligned by 15 arcsecs, the error introduced is below 0.002% of the angle being measured, for angles up to ±10°. This error is far smaller than both the ±0.2% accuracy specification of standard Angular optics, and the ±0.02% accuracy specification of calibrated Angular optics. Therefore this error is usually ignored.

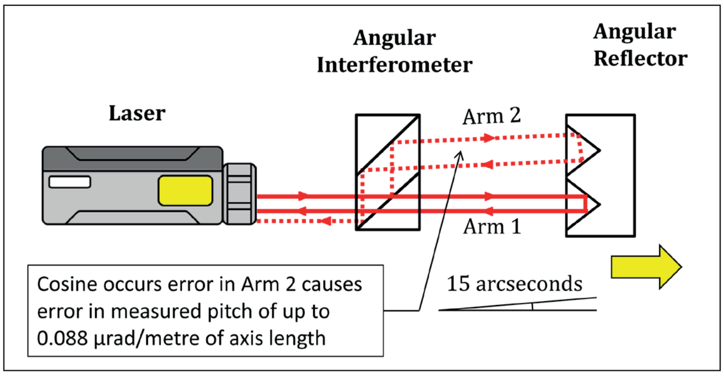


Figure 18

Secondly, consider the effect when using the optics to measure the pitch error of a linear axis as shown in Figure 18. Assume that the axis is free from any pitch error. The beams in Arm 1 are shown perfectly aligned, and the beams in Arm 2 are shown misaligned by the worst case periscope tolerance of 15 arcsecs. The misalignment of the beams in Arm 2 will cause a cosine error in its length so, as the reflector is moved along the axis, (in the direction of the yellow arrow), the path lengths of Arms 1 and 2 will not increase at the same rate. This will cause a small but steady accumulation in the pitch angle readout of 0.088 µrad (0.018 arcsec) for every metre travelled along the linear axis. It is this misalignment that is responsible for the ±0.1 Mµrad term in the accuracy specification of Renishaw's standard angular optics (M is the linear measurement distance between the angular interferometer and the angular reflector in metres). The error shown in this example is relatively insignificant.

However, the size of the error also depends on the overall accuracy of alignment of the laser beam to the linear axis.

Suppose the laser head is adjusted so that Arm 1 was misaligned by -7.5 arcsecs, then Arm 2 will now be misaligned by +7.5 arcsecs. The cosine error in both beams will then be identical, and the measurement error disappears.

But, if the laser head is adjusted so that Arm 1 is misaligned by +15 arcsecs, then Arm 2 will now be misaligned by +30 arcsecs. Both beams will now see a cosine error in measurement, but that seen by Arm 2 will be larger. Because the cosine error increases as the square of the misalignment angle, the original error of 0.088 µrad/m of axis travel is trebled* to 0.264 µrad/m (0.054 arcsec/m).
($15^2-0^2 = 225$, $30^2-15^2 = 675 = 3 \times 225$*).*

The error is still relatively small, but indicates that alignment of the laser is important. On long axes, accurate alignment is easily achieved by eye and the signal strength meter. But, as the axis length gets shorter, accurate alignment becomes more difficult. If the user aligns the laser head such that the beams in Arms 1 and 2 remain aligned within 0.5 mm to the retro-reflectors as they move along the full length of the axis (i.e. return beams aligned within 1 mm), the worst case pitch measurement error that will accumulate along the full length of the axis is shown in Figure 19.

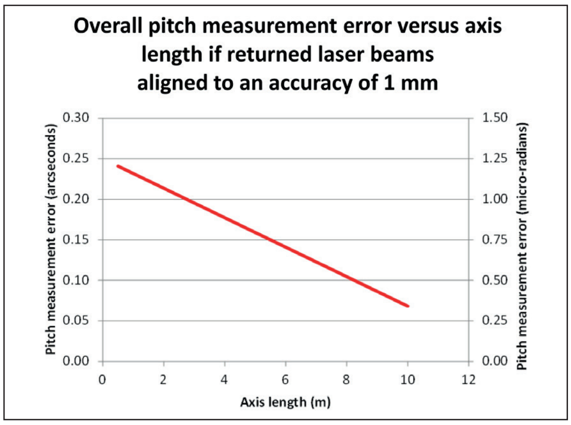


Figure 19

On short axes the measurement error can be reduced, if necessary, by auto-reflection using a slip block or mirror placed against a machine surface that is perpendicular to the axis of motion whilst adjusting the alignment of the laser.

Thermal expansion of angular interferometer periscope

Examination of the beam paths through the interferometer (see Figure 20) shows that there is an imbalance in the path lengths of Arms 1 and 2 of the interferometer. In Renishaw's standard angular optics, Arm 2 travels through more glass than the beam in Arm 1. If the temperature changes, after the reading is datumed, this imbalance will cause a change in the path length of Arm 2 which is not balanced by a corresponding change in the path length of Arm 1. This will cause a change in the angular reading.

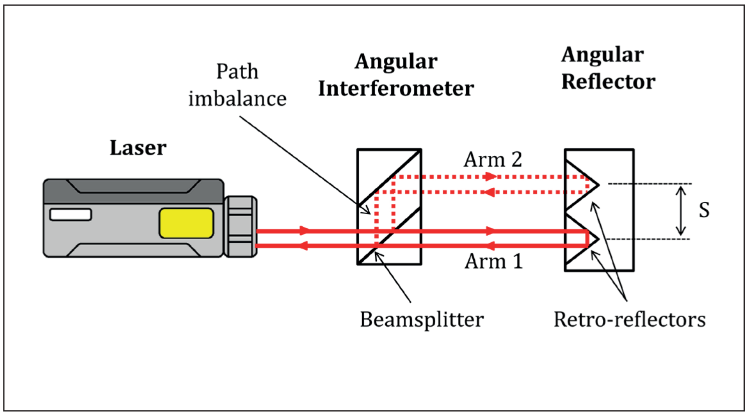


Figure 20

For Renishaw's standard angular optics, the change in angular reading, $\Delta\theta$, is given by the following equation and values:

$$\Delta\theta = \Delta T(\alpha.n + \delta n/\delta T)$$

Where:-

$\Delta\theta$ = change in reading (μrad)
ΔT = change in interferometer glass temperature since datum (°C)
$\alpha = 7.1$ = linear coefficient of expansion of the glass (ppm/°C)
$n = 1.51872$ = refractive index of glass
$\delta n/\delta T = 3$ = temperature coefficient of refractive index n (ppm/°C)

Substituting the values shown above gives a temperature sensitivity of 13.8 μrad/°C or 2.8 arcsec/°C.

The best way to minimise errors from this source are as follows:

- Ensure the optics have thermally acclimatised to the local environment before starting to take readings. *Note: Renishaw's standard Aluminium housed angular optics will acclimatise much more quickly than steel housed optics due to their higher thermal conductivity, lower thermal mass and black surface finish.*
- Minimise temperature variations in the local environment whilst taking readings. Keep doors closed, avoid direct sunlight, etc. It may be beneficial to drape a cloth over the optics housing to thermally insulate them from local temperature changes. *Note: Renishaw's steel housed angular optics are slightly less sensitive to small short term temperature fluctuations than Aluminium housed optics, since the higher thermal mass and lower conductivity of steel will help dampen out such fluctuations.*
- Take readings swiftly after datuming. Re-datum if necessary.

Note: Although expansion of the periscope increases the outward beam separation between Arms 1 and 2 of the interferometer, the action of the retro-reflectors will reduce the separation of the returned beams by an equal and opposite amount. Therefore the average beam separation (S) is unaffected and is determined solely by the retro-reflector spacing and not the periscope geometry.

Error due to lack of air refractive index compensation

The laser's wavelength in air is dependent on the air's refractive index which varies slightly with air temperature, pressure and humidity. These will vary according to the weather and altitude above sea level. If the air's refractive index rises the laser wavelength in air is reduced. Therefore, if the environment changes, the apparent path lengths (as measured by the laser) of Arms 1 and 2 will be altered slightly. If the reflector and interferometer are nearly parallel (such as at datum, or when measuring small pitch or yaw angles, the lengths of Arms 1 and 2 that are in air are almost identical. Therefore, any likely change in refractive index will affect both arms almost identically and the effect on the angular reading is negligible.

However, when measuring large angles, the air path lengths of Arms 1 and 2 may differ by 10 mm or so, making the angular reading more sensitive to air refraction changes.

Variation in refractive index of air under extremes of atmospheric conditions at sea level			
Environment	STP	+ve extreme	-ve extreme
Air Pressure (mBar)	1,013	900	1100
Air Temperature (°C)	20	40	0
Humidity (%RH)	51	100	0
Analysis			
Air refractive index	1.0002714	1.0002234	1.0003167
Change versus stp (ppm)	0	48	-45

Figure 21

The table in Figure 21 shows the worst case changes in air refractive index that may occur due to variations in the environment caused by local weather conditions. (The variation is shown in ppm relative to standard sea level atmospheric conditions of 1013.25 mBar, 20°C, 50%RH with the temperature variation restricted to the 0-40°C operating temperature range of the Renishaw laser). The table shows that, if variations in air refractive index are not compensated for, even under the most extreme weather variations, the error in angular measurement will be less than ±50ppm (0.005%) of the measured angle.

Variation in air pressure with altitude above sea level at 20 deg C, 50%RH, and the effect on refractive index			
Altitude (m)	Pressure (mbar)	Refractive index n	Change (ppm)
0	1013.25	1.0002714	0
500	954.61	1.0002557	-16
1000	898.75	1.0002407	-31
1500	845.56	1.0002264	-45
2000	794.95	1.0002128	-59
2500	746.83	1.0001999	-71
3000	701.09	1.0001877	-84
3500	657.64	1.0001760	-95
4000	616.40	1.0001649	-106
4500	577.28	1.0001544	-117
5000	540.20	1.0001445	-127

Figure 22

However, atmospheric pressure also falls with increasing by altitude. The table in Figure 22 shows how pressure falls with increasing altitude and how this alters the air's refractive index. Since most interferometer systems are used below 2500m, the table shows that, if air refractive index variations are not compensated for, the error in angular measurement due to altitude will be less than about 70ppm (0.007%) of the measured angle. However, in extreme locations (e.g. mountain top telescopes at 5000m) the error may approach 130ppm (0.013%) of the measured reading.

It is clear that the angular measurement error introduced by not compensating for refractive index changes due to weather (0.005%) and altitude (0.013%) are insignificant compared to the accuracy specification of Renishaw's standard ±0.2% accuracy angular optics, and within the ±0.02% uncertainty of calibrated angular optics. It is for this reason that Renishaw's angular software does not apply air refraction compensation to angular readings (it uses the refractive index of air under standard conditions). This has the benefit of allowing angular measurements to proceed without requiring the use of a weather station or manual entry of environmental conditions.

Notes:

1) *When angular optics are used in combination with XR20-W and Renishaw's rotary axis calibration software, the angular factor derived from the automatic calibration cycle of the optics takes into account the local air refractive index, thereby eliminating any significant air refraction errors due to altitude or local weather conditions.*
2) *If using calibrated angular optics to measure large angles without XR20-W under extreme environmental conditions (e.g. high altitude) the angular factor, K, may be adjusted to reduce the measurement error, if required.*

Misalignment of angular reflector to axis of rotation/measurement

If the angular reflector is not accurately aligned so that the centre-line through the retro-reflectors is perpendicular to the axis of the rotational error being measured (refer to Figure 23), then sensitivity to rotation about an axis perpendicular to the axis of measurement will be introduced whilst sensitivity to rotation about the desired axis of measurement will be very slightly reduced.

In the example shown in Figure 23, suppose the angular optic is being used to measure the pitch error in the motion of the x axis of a machine. The correctly aligned optic (shown on the left) will correctly measure rotations about the y axis (i.e. pitch errors in the x axis of motion) and will be insensitive to rotations about the z axis (i.e yaw errors in the x axis motion).

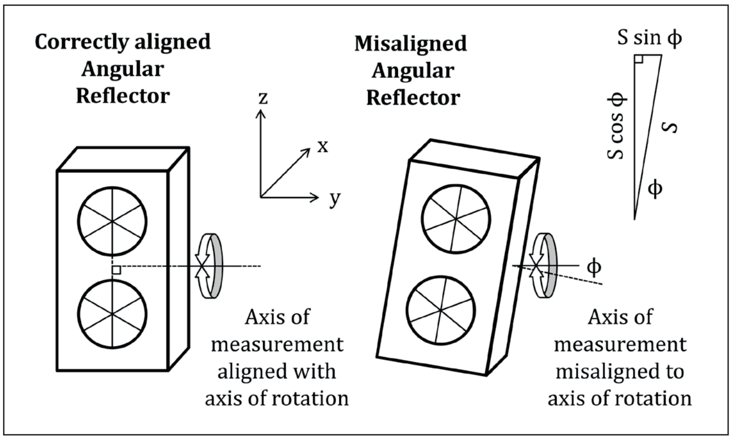


Figure 23

However, if the optic is misaligned (as shown on the right) by an angle φ, then the effective separation between the retro-reflectors, in the desired plane of measurement, is reduced from S to S.cos(φ), thereby reducing sensitivity to rotations about the y axis (i.e. pitch) errors slightly. Similarly a small misalignment of S.sin(φ) is introduced at 90° to desired plane of measurement, this causes some sensitivity to rotations about the z axis (i.e. yaw errors) in the axis of motion.

Suppose the optic misalignment, φ, is 1°. Misalignments of up to this level are not unusual, when aligning the optics by eye, but larger misalignments will cause noticeable misalignment of the housings and returned laser beams.

Firstly consider the accuracy of the pitch measurement with 1° misalignment. The angular measurement will indicate 99.985% of the pitch error in the x axis. This is an error of -0.015% which is negligible when using ±0.2% accuracy standard angular optics when measuring axis pitch or yaw errors. However, when measuring large angles (such as when calibrating a rotary axis) the error is significant. Fortunately, when using XR20-W with Renishaw's rotary axis calibration software, the error is eliminated by the automatic optics calibration cycle which will adjust the angular factor accordingly.

Now consider the sensitivity to yaw errors. Again, assuming the optic misalignment is 1°, the sensitivity to yaw error that will be introduced is 1.7%. So, with this degree of misalignment, the angular readout will indicate 99.985% of the pitch error in the axis, and 1.7% of the yaw error. i.e. there is a small degree of "cross-contamination" of the pitch error readout by any yaw error in the axis. Typically, machines have small and often similar amounts of pitch and yaw errors in their linear axes, so this error is ignored. However, if the yaw error is much larger than the pitch error the cross-contamination would be more significant. If there are concerns, it is advisable to check both the pitch and yaw errors of the axis.

When calibrating a rotary axis with XR20-W the mechanical tolerances of the optics housings, the indexer and the mounting hardware ensures the centreline between the retro-reflectors is accurately aligned relative to the axis of rotation of XR20-W, and its mounting plate. However, it is important that the axis of rotation of the XR20-W is accurately aligned, (in angular terms), relative to the rotary axis under test. This is easy if the XR20-W is being mounted on an axis with a suitably accurate surface (e.g. rotary table). If in doubt a clock gauge can be used to verify the suitability of the mounting surface, before mounting the XR20-W. This topic is covered in more detail in technical white paper TE327 *'Interferometic calibration of rotary axes'.*

When measuring surface flatness with angular optics it is common for straightness errors in the straight-edge, which guides the angular reflector, to be larger than the flatness error in the surface being measured. If the straight-edge is not straight, it will cause the reflector to yaw as moves along the measuring line. If the optic is misaligned, this yaw error will contaminate the pitch measurements that are used to calculate surface flatness. Fortunately the mechanical tolerances of the angular reflector and flatness bases will typically ensure the centre-line between the retro-reflectors is perpendicular to the table surface within about 0.25°, reducing the cross-contamination to below 0.5%. Nevertheless, it is good practice to use a high quality straight-edge and to check the feet on the flatness base have not been damaged or picked up swarf etc. before use.

Fringe sub-division errors

As mentioned previously, as the angular reflector rotates, the relative change between the path lengths in Arms 1 and 2 is detected by an interference fringe counter/interpolator inside the laser's detector unit and then converted into a linear distance ΔL by multiplying by the laser wavelength/2.

$$\Delta L = \text{Fringe count} \times \text{laser wavelength} / 2$$

The laser system software then converts the relative change in path lengths, ΔL, into an angle θ by calculating $\theta = \Delta L/S$ (or $\theta = \arcsin(\Delta L/S)$ if arcsine correction is enabled). The angular resolution available, using standard 30 mm spaced optics, by simply using a whole number fringe count would only be about ½ (633 nm/30 mm) ≈ 10 µrad (≈ 2 arcsecs), which is insufficient for high accuracy angular measurements. Fortunately the laser's detector unit also includes a fringe interpolator, which can divide each fringe into 256 sub-divisions. This increases the resolution to approximately 0.04 µrad (0.01 arcsec). However, the sub-division process is not perfect. Ideally the 256 interpolated sub-divisions would all be exactly equal in size. But, due to small optical and electrical imperfections, they aren't. This causes a small, periodic (sinusoidal) measurement error of up to ±0.33 µrad (less than 0.07 arcsec) to be superimposed on to the measurements taken. This sub-divisional error is included within the ±0.5 µrad (±0.1 arcsec) error terms in the accuracy specification of Renishaw's standard accuracy and calibrated angular optics. The periodic error repeats once or twice for every 2 arcsecs.

A typical form of this error is illustrated graphically in Figure 24. The dotted line shows the relationship between true and measured angles for a perfect system (with no sub-divisional error). The solid line shows the typical relationship between true and measured angle for a system with a ±0.07 arcsec sub-divisional error.

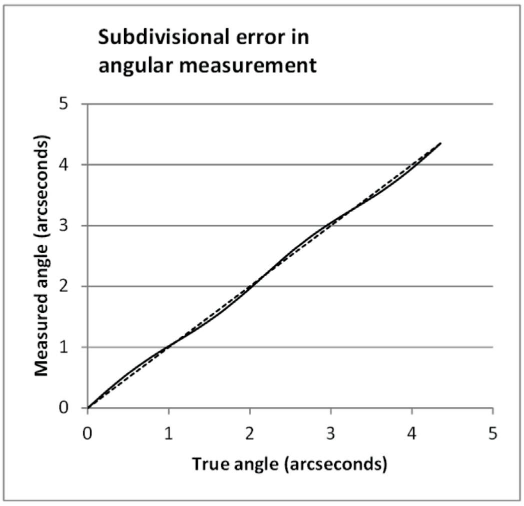


Figure 24

This small error is typically ignored. However, in demanding applications there are various steps that can be taken to minimise this error.

- Maximise the signal strength (although do not use the high gain setting on XL-80 unless necessary, since this may cause larger errors due to clipping of the electrical signals)
- Ensure the angular interferometer is carefully aligned within ±1° in roll pitch and yaw, relative to the laser head. (A misaligned interferometer can cause some cross mixing of the polarisations of the light within Arms 1 and 2 of the interferometer, increasing sub-divisional errors). Alignment can be checked by placing a reflective slip block or mirror against the interferometer optic housing to auto-reflect the laser beam back to the output aperture whilst adjusting the pitch and yaw, and roll alignment maybe checked with a small spirit level.
- In the most demanding applications, it is possible to use an interferometer and reflector with a much larger spacing between Arms 1 and 2. For applications within National Standards Laboratories Renishaw has supplied special optics with a beam separation of 150 mm (5 times wider than usual). This reduces the sub-divisional error by a factor of 5, to below 0.02 arcsec, albeit at the expense of ease of use, portability, reduced angular measurement range and cost.

This completes the review of the main error sources that affect the accuracy of interferometric angle measurement.

Angular measurement hardware options available from Renishaw

Renishaw can supply a variety of angular interferometer components and options which provide a choice of accuracy levels, beam spacing and housing material. Some of these options are subject to availability. The combinations that are available are shown in Figure 26 at the end of this section, together with a comparative performance chart.

Accuracy levels

Renishaw supply angular optics in two accuracy grades:

- Standard ±0.2%. For 30 mm beam spaced optics the uncertainty of measurement is:
  - $\pm 0.2\%\theta \pm 0.5 \pm 0.1M$ µrad
- Individually calibrated ±0.02% and supplied with a calibration certificate. For 30 mm beam spaced optics, the uncertainty of measurement (dependent on the environment) is:
  - $\pm 0.02\%\theta \pm 0.5 \pm 0.1M$ µrad

Where θ is the angle measured in µrad and M is the linear measurement range between the angular interferometer and angular reflector in metres.

The first terms (i.e. $\pm 0.2\%\theta$ and $\pm 0.02\%\theta$) in the specifications above are predominantly caused by the tolerance on the separation between the retro-reflectors in the angular reflector, and can be reduced by selecting optics with a higher manufacturing tolerance, or by calibrating the actual separation between the retro-reflectors, and entering a revised angular factor into the software.

The second term (i.e. ±0.5 µrad) is an additional error in µrad which is added on top of the first term. This error is predominantly caused by fringe sub-division error. The error can be reduced by selecting angular optics with a larger separation between the retro-reflectors in the angular reflector.

The third term (i.e. ± 0.1M µrad) is an additional ±error in µrad which is added on top of the first two terms. This term increases with the measuring range M such as when measuring pitch or yaw errors in a linear axis. M is expressed in metres. This error is predominantly caused by the tolerance on the non-parallelism of the beams emerging from the angular periscope.

Housing materials

Renishaw provide 3 options for housing materials:

- Aluminium - Ideally suited to the majority of applications where speed of use*, portability and cost are major factors. **Aluminium housed optics acclimatise much more quickly than stainless steel or invar due to the higher thermal conductivity and lower thermal mass of aluminium. This speed of acclimatisation saves valuable time when moving from calibrating one machine to another.*

- Stainless Steel - Has a lower coefficient of thermal expansion (10ppm/°C) than Aluminium (23ppm/°C). But it also has a much lower thermal conductivity than Aluminium and is much heavier and so takes longer to acclimatise to a change of environment. Stainless steel housed optics are best suited to use in a fixed location where portability is not a concern and the increased thermal inertia and stability may help to dampen out the effects of small oscillations in ambient temperature.

- Invar - Chosen for its low thermal expansion coefficient (1.2ppm/°C). An Invar reflector housing may be selected to improve the stability of the spacing between the angular retro-reflectors in both 30 mm and 150 mm beam spaced designs (see below). An Invar interferometer/periscope housing also improves the stability of the 150 mm beam spaced design (see below).

Beam spacing

There are 2 basic designs

- 30 mm beam spacing (standard)
- 150 mm beam spacing. This design (see Figure 25) has been supplied as a special to National Standards Laboratories. The housings are made from Invar for maximum thermal stability. The design ensures that the path lengths of Arms 1 and 2, within glass, are almost identical and the spacing of the periscope is determined by invar rather than glass, thereby largely eliminating any drift in angular reading due to thermal expansion of the periscope. The main optical path imbalance between the arms of the interferometer occurs in air, which has a very low thermal coefficient of just 1ppm/°C. The 150 mm beam spacing is 5 times greater than the standard spacing of 30 mm, providing the system with 5 times greater resolution and a 5 times reduction in the sub-divisional error. The angular measurement range is also significantly reduced. It should be noted that these optics are not compatible with XR20-W and Renishaw's Angular and Rotary Axis Calibration software. Their use is highly specialised and usually requires application specific software written using Renishaw software development kit. The accuracy specification of these optics, when used for rotary axis calibration is:

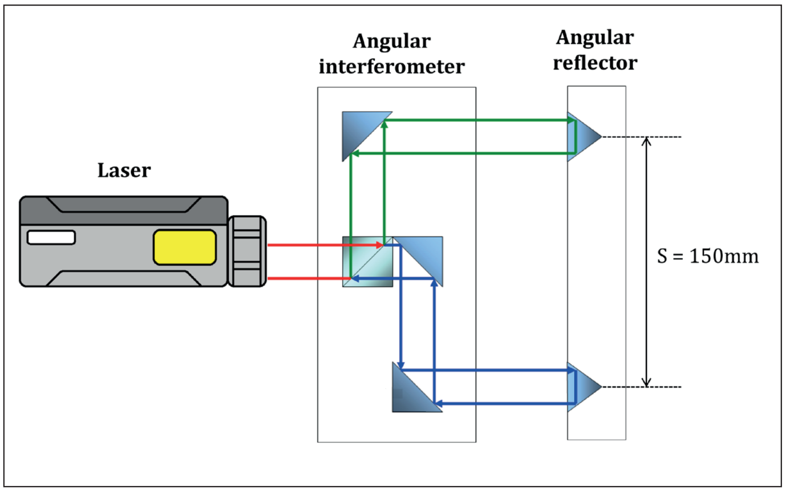


Figure 25

  - ±0.005%θ ±0.1 µrad

  Note the third accuracy term (±0.1 M) is omitted because these optics are not designed for pitch or yaw measurement.

Performance table

The table in Figure 26 shows relative performance ratings (1-5 stars) for each of the different designs available from Renishaw against a variety of performance indicators.

Relative performance ratings against a variety of performance criteria					
Beam spacing	**30mm**				**150mm**
Accuracy grade	**Std ±0.2%**	**Calibrated ±0.02%**			
Housing material	**Aluminium**	**Aluminium**	**Steel**	**Invar (30mm)**	**Invar (150mm)**
Small angle accuracy (note 1)	☆☆☆☆	☆☆☆☆	☆☆☆☆	☆☆☆☆	☆☆☆☆☆
Large angle accuracy - without XR20-W (note 2)	☆☆	☆☆☆☆	☆☆☆☆	☆☆☆☆	☆☆☆☆☆
Large angle accuracy - with XR20-W	Note 3	Note 3	Note 3	Note 3	Note 4
Angular measuring range	☆☆☆☆☆	☆☆☆☆☆	☆☆☆☆☆	☆☆☆☆☆	☆☆
Use with standard Renishaw software	☆☆☆☆☆	☆☆☆☆☆	☆☆☆☆☆	☆☆☆☆☆	Note 4
Portability	☆☆☆☆☆	☆☆☆☆☆	☆☆☆	☆☆☆	☆
Thermal stability	☆☆	☆☆	☆☆☆	☆☆☆☆	☆☆☆☆☆
Speed of thermal acclimatisation	☆☆☆☆☆	☆☆☆☆☆	☆☆	☆☆☆☆☆	☆☆☆
Suitability for travelling service/calibration	☆☆☆☆☆	☆☆☆☆☆	☆☆☆	☆☆☆	☆

Notes: 1) Applies when measuring small angles (<100 arcsec) such as axis pitch and yaw
2) Applies when measuring larger angles (>100 arcsec) without XR20-W
3) XR20-W contains an integrated aluminium housed angular reflector.
The fully automatic XR20-W 'in situ' optics calibration cycles ensures 5 star accuracy
4) Incompatible

Figure 26

Conclusion

This paper has described the operational principles behind Renishaw's interferometric angular optics and how they can be used to measure pitch or yaw errors in a linear axis, the angular accuracy of a rotary axis or surface flatness. It has also examined various error sources that can affect the accuracy of measurement and described the various angular optics options available from Renishaw and how they have been designed to address these error sources, concluding with a relative performance table.

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